Plasma electron kinetics in a weak high-frequency field and magnetic field amplification

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We describe the linear stage of Weibel instability in a plasma heated via inverse bremsstrahlung absorption of a high-frequency, moderate intensity radiation field under conditions in which the plasma electron velocity distribution function is weakly anisotropic. We report on the possibility of a significant amplification of spontaneous magnetic fields both in the case of an electron distribution function slightly departing from a Maxwellian in the region of subthermal velocities, and in the case where the Langdon nonequilibrium distribution is formed. We show that the direct influence of collisions on the Weibel instability growth rate may be traced back to subthermal electrons, for which the effective collision frequency is large.

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I. INTRODUCTION

Collisions of electrons with ions are one of the main physical processes giving rise to absorption of electromagnetic radiation and to plasma heating (see, for instance, Ref. [1], and literature quoted there). This process is especially important in a hot plasma with multicharged ions, when $Z > 1$, Z being the ion ionization multiplicity. For the case where the amplitude of electron quiver velocity v_E is smaller than the electron thermal velocity v_T, many papers [2–13] have been devoted to the investigation of the inverse bremsstrahlung peculiarities and of the kinetics of the heated electrons. Generally, in such a case, special attention has been paid to the isotropic part of the distribution function. At the same time, it must be added that even a weak radiation field is able to give rise to anisotropy (in velocity space) in the distribution function, which may be the cause leading to the development of various electromagnetic instabilities (see, for instance, Refs. [14,15]). In particular, as shown in Refs. [16–18], anisotropy of electron distribution at inverse bremsstrahlung of radiation is the cause of Weibel instability.

In Ref. [17] the possibility of development of the Weibel instability has been demonstrated for the case where $v_E \gg v_T$. Such conditions are of interest when a rather intense radiation field interacts with a not too hot plasma. We observe also that in the present work, similarly to Refs. [16–18], we do not consider the case where v_E or v_T is comparable to the speed of light, which would require a relativistic treatment. Another (but rather ubiquitous in this kind of treatment) limitation is the assumption that both the plasma and the radiation exhibit no inhomogeneities. To realize similar conditions is relatively simple if the radiation frequency significantly exceeds the electron plasma frequency. At the same time, for real experiments, the cases where v_E is smaller than or comparable to v_T, still with $v_T \ll c$, are also of interest. A large number of papers concerned with laser-plasma interactions are devoted to just such cases (see, for instance, Refs. [3–7,9–13,16,18]). It is the aim of this work to show that in such conditions the generation of strong quasistationary magnetic fields is possible, which may substantially affect the kinetic and electromagnetic phenomena in the plasma. With papers [16–18] in mind, in this work we consider the linear stage of the Weibel instability development, on the basis of a detailed description of the electron kinetics in the presence of a high-frequency field. The possibility of avoiding discussion of a nonlinear stage of the Weibel instability exists considering short-pulsed electromagnetic radiation. It is understood that the existence of such a possibility is by no means to be interpreted as unique, excluding other conditions in which the nonlinear stage of the Weibel instability may develop in the presence of an ultrashort and very strong laser pulse. In particular, this possibility has been demonstrated in Ref. [17]. Further, we demonstrate that it is possible to considerably amplify a spontaneous magnetic field in a plasma, when the electron distribution function is close to a quasistationary modified Maxwellian distribution, or when it evolves towards the nonequilibrium Langdon distribution. We demonstrate that, due to the relatively lower number of subthermal electrons characteristic of the Langdon distribution (as compared to a Maxwellian), the Weibel instability growth rate is smaller and the efficiency of a magnetic field amplification is decreased. Our analysis shows that it is necessary to take into account the influence of electron collisions not only to determine the shape of the distribution function, but also to correctly derive the Weibel instability growth rate. In particular, such an influence causes relative weakening of the magnetic field amplification efficiency. This statement is, to some extent, similar to that on how collisions affect the Weibel instability [20], with, however, an essential difference. In the conditions under consideration here, the effective electron collision frequency, responsible for the decrease of the instability growth rate during inverse bremsstrahlung, may be controlled by subthermal electrons.

The paper is organized as follows. In Sec. II we give detailed information on the electron distribution function in a plasma heated by high-frequency electromagnetic radiation. In Sec. III, formulas for the Weibel instability growth rate are derived. The formulas take into account the direct influence of electron-ion collisions on the growth rate. In Sec. IV we consider vortex magnetic field amplification in a plasma with electron distribution close to a perturbed Maxwellian distribution. In Sec. V similar consideration is given to the case where an initially Maxwellian distribution evolves to the Langdon distribution. Finally in Sec. VI an estimate of the generated magnetic field strength is given and the possibility of magnetic field influence on the electron transport is demonstrated.

II. ELECTRON DISTRIBUTION FUNCTION

Let us consider a fully ionized plasma in the presence of the high-frequency field,

$$\vec{E}(t)=\frac{1}{2}\vec{E}\exp(-i\omega_0 t)+\text{c.c.}, \tag{1}$$

where $\vec{E}=(0,0,E)$, and the frequency ω_0 significantly exceeds the electron effective collision frequency. The field strength will be assumed such that the quiver velocity $v_E=|eE/m\omega_0|$ is small compared to the characteristic velocity of the electrons taken into consideration. Confining the analysis to the case when Z is high $(Z>1)$, for the main part of the electron distribution function weakly varying over the field period, to the second order of the perturbation theory in the field strength, we have the kinetic equation (see, for instance, Ref. [16])

$$\frac{\partial f}{\partial t}+\frac{1}{4}\left\{\left(\vec{v}_E\frac{\partial}{\partial \vec{v}}\right)\cdot \text{St}\left(\vec{v}_E^*\frac{\partial f}{\partial \vec{v}}\right)+\left(\vec{v}_E^*\frac{\partial}{\partial \vec{v}}\right)\text{St}\left(\vec{v}_E\frac{\partial f}{\partial \vec{v}}\right)\right\}=\text{St}(f)+\text{St}(f,f), \tag{2}$$

here St(f) and St(f,f) are, respectively, the electron-ion and the electron-electron collision integrals having the forms

$$\text{St}(f)=\frac{1}{2}\nu(v)\frac{\partial}{\partial v_\alpha}(v^2\delta_{\alpha\beta}-v_\alpha v_\beta)\frac{\partial f}{\partial v_\beta} \tag{3}$$

and

$$\text{St}(f,f)=\frac{1}{2nZ}\frac{\partial}{\partial v_\alpha}\int d\vec{v}'\,\nu(|\vec{v}-\vec{v}'|)[(|\vec{v}-\vec{v}'|)^2\delta_{\alpha\beta}-(v_\alpha-v'_\alpha)(v_\beta-v'_\beta)]\left(\frac{\partial}{\partial v_\beta}-\frac{\partial}{\partial v'_\beta}\right)f(\vec{v},t)f(\vec{v}'t), \tag{4}$$

$f=f(\vec{v},t)$, $\nu(v)$ is the electron-ion collision frequency

$$\nu(v)=\frac{4\pi Ze^4n\Lambda}{m^2v^3}, \tag{5}$$

n is the electron density, and Λ is the Coulomb logarithm.

Assuming a weak influence of the high-frequency field, the electron distribution function is written as $f=f_0+\delta f$, where $f_0=(1/4\pi)\int f\,d\Omega$ is the isotropic part with $d\Omega$ the solid angle element of the velocity vector, and $|\delta f|\ll f_0$ is a small addition.

Using the above condition and averaging Eq. (2) over the velocity directions, for $f_0=f_0(v,t)$ we obtain the equation

$$\frac{\partial f_0(v,t)}{\partial t}=\frac{v_E^2}{6v^2}\frac{\partial}{\partial v}\left[v^2\nu(v)\frac{\partial f_0(v,t)}{\partial v}\right]+\frac{4\pi}{3nZ}v\nu(v)\times\frac{\partial}{\partial v}\left\{\left[v^2\int_v^\infty dv'v'f_0(v',t)+\frac{1}{v}\int_0^v dv'v'^4f_0(v',t)\right]\frac{\partial f_0(v,t)}{\partial v}+3f_0(v,t)\int_0^v dv'v'^2f_0(v',t)\right\}. \tag{6}$$

Solutions to Eq. (6) are known for two limiting cases. If Langdon parameter is small

$$\alpha=\frac{Zv_E^2}{v_T^2}\ll 1, \tag{7}$$

v_T being the electron thermal velocity, a relatively slow electron heating takes place due to inverse bremsstrahlung made possible by the electron-ion collisions. In the limit $\alpha\ll 1$ in Eq. (6) it is possible to neglect the slow variations with time of the function f_0. In such conditions a solution of Eq. (6) is the quasistationary distribution [13]

$$f_0(v,t)=\frac{n}{(2\pi)^{3/2}v_T^3}\exp\left[-\frac{1}{v_T^2}\int_0^v\frac{u^4}{u^3+v_L^3}du\right], \tag{8}$$

where $v_L=v_T(\alpha\sqrt{\pi/8})^{1/3}<v_T$, and the thermal velocity v_T increases with time due to electron heating according to equation

$$\frac{d}{dt}v_T^2=\frac{2}{9\sqrt{2\pi}}\nu v_E^2, \tag{9}$$

where $\nu=\nu(v_T)$ is the effective collision frequency of thermal electrons. In the region of high velocities $(v\gg v_L)$, the distribution (8) goes over to a Maxwellian. When, instead, one has $v\lesssim v_L$, f_0 depends on velocity as in the case when a sufficiently rapid electron heating takes place; namely, $f_0\sim\exp(-u^5/5v_T^2v_L^3)\cong 1-u^5/5v_T^2v_L^3$. In the opposite limiting case, when $\alpha\gg 1$, in Eq. (6), we can neglect the influence of a relatively infrequent electron-electron collisions on the distribution function. In such a case, after the characteristic time roughly required for the doubling of the electron initial temperature $t\gtrsim v_T/\nu v_E^2$ the following self-similar distribution forms [4,5]

$$f_0(v,t)=\frac{5n}{12\pi\sqrt{3}v_T^3(t)}\left[\Gamma\left(\frac{3}{5}\right)\right]^{-5/2}\times\exp\left\{-\frac{v^5}{9\sqrt{3}v_T^5(t)}\left[\Gamma\left(\frac{3}{5}\right)\right]^{-5/2}\right\}, \tag{10}$$

where $\Gamma(3/5)$ is the gamma function, and the effective thermal velocity increases with time according to equation

$$v_T^4(t)\frac{d}{dt}v_T(t)=\frac{5}{162}\sqrt{3}\left[\Gamma\left(\frac{3}{5}\right)\right]^{-5/2}\nu(v_E)v_E^5. \qquad (11)$$

Next, subtracting Eq. (6) from Eq. (2) we obtain a linear equation for the small anisotropic addition to the isotropic distribution

$$\frac{\partial}{\partial t}\delta f-St(\delta f)=\frac{1}{2}v_{E\alpha}v_{E\beta}^{*}\left(v_\alpha v_\beta-\frac{1}{3}v^2\delta_{\alpha\beta}\right)\times\frac{1}{v}\frac{\partial}{\partial v}\left[\frac{\nu(v)}{v}\frac{\partial f_0}{\partial v}\right]. \qquad (12)$$

As $Z>1$, in the left hand side of Eq. (12) the electron-electron collision integral has been omitted. For times greater than the inverse of the collision frequency, when $\nu(v)t\gg 1$, the quasistationary solution to Eq. (12) results in

$$\delta f=\frac{1}{6}\left\{(\vec{v}_E\vec{v})(\vec{v}_E^{*}\vec{v})-\frac{1}{3}v^2v_E^2\right\}\frac{1}{v\,\nu(v)}\frac{\partial}{\partial v}\left[\frac{\nu(v)}{v}\frac{\partial f_0}{\partial v}\right]. \qquad (13)$$

The relation (13), together with the explicit expressions (8)–(11) for the isotropic part of the distribution function reported above, form the basis of the analysis given below of the stability of the plasma state in the presence of a high frequency field.

III. GROWTH RATE OF WEIBEL INSTABILITY

It is well known [14,15] that anisotropic electron distributions are unstable against the development of the Weibel instability when the perturbations of the electromagnetic field and of the distribution functions have the spatial dependency

$$\delta\vec{E},\quad \delta\vec{B},\quad \delta F\sim\exp(i\vec{k}\cdot\vec{r}). \qquad (14)$$

In a linearly polarized field directed along the z axis the electromagnetic field perturbations most effectively excited have the configuration $\vec{k}=(k,0,0)$, $\delta\vec{E}=(0,0,\delta E)$, $\delta\vec{B}=(0,\delta B,0)$. Such perturbations are described by the following system of Maxwell equations:

$$\frac{\partial}{\partial t}\delta B=ikc\,\delta E, \qquad (15)$$

$$\frac{\partial}{\partial t}\delta E=ikc\,\delta B-4\pi e\int d\vec{v}\,v_z\delta F, \qquad (16)$$

and by the kinetic equation for the distribution function perturbation

$$\frac{\partial}{\partial t}\delta F+ikv_x\delta F-St(\delta F)=-\frac{e}{m}\delta E\frac{\partial}{\partial v_z}f+\frac{e}{mc}\delta B\left(v_z\frac{\partial}{\partial v_x}-v_x\frac{\partial}{\partial v_z}\right)f, \qquad (17)$$

where c is the speed of light. We look for a solution of equations (15)–(17) of the form

$$\delta E,\quad \delta B,\quad \delta F\sim\exp\left[\int_0^t dt'\,\gamma(t')\right], \qquad (18)$$

where $\gamma(t)$ is the time-dependent growth rate of the instability.

Assuming that near the threshold for the onset of Weibel instability for the bulk of the electrons the following inequalities are fulfilled:

$$kc\gg k|v_x|\gg\nu(v),\quad \gamma(t), \qquad (19)$$

from Eqs. (15)–(17) the equation for the instability growth rate $\gamma=\gamma(t)$ is obtained:

$$-\gamma\frac{\pi}{n}\int d\vec{v}\,\delta(kv_x)\frac{v_z^2}{v_x}\frac{\partial f}{\partial v_x}=\frac{1}{n}\int d\vec{v}\,v_z\left\{\left(\frac{\partial}{\partial v_z}-\frac{v_z}{v_x}\frac{\partial}{\partial v_x}\right)f+\pi\delta(kv_x)\times St\left(\frac{\partial f}{\partial v_z}-\frac{v_z}{v_x}\frac{\partial f}{\partial v_x}\right)\right\}-\frac{k^2c^2}{\omega_L^2}, \qquad (20)$$

where $\delta(kv_x)$ is the delta function and $\omega_L=(4\pi e^2n/m)^{1/2}$ is the electron plasma frequency. Further, taking into account the relations (3) and (13), assuming $f=f_0+\delta f$, from Eq. (20) we obtain

$$-\gamma\frac{\pi^2}{kn}\int_0^\infty dv\,v^2\frac{\partial f_o}{\partial v}=-\frac{8\pi}{3n}v_E^2\int_0^\infty dv\,v\frac{\partial f_o}{\partial v}\left[1-\frac{\pi}{4}\frac{\nu(v)}{kv}\right]-\frac{k^2c^2}{\omega_L^2}. \qquad (21)$$

According to Eq. (21) the explicit form of the growth rate γ depends on the shape of the electron distribution f_0, which results from the solution to Eq. (6). When Langdon parameter (7) is small and f_0 is given by expression (8), from Eq. (21) one has

$$\gamma=\sqrt{\frac{2}{\pi}}kv_T\left\{\frac{2}{3}\frac{v_E^2}{v_T^2}-\frac{k^2c^2}{\omega_L^2}-\frac{\pi\sqrt{2\pi}}{9\sqrt{3}}\frac{v_E^2}{v_T^2}\frac{\nu}{kv_L}\right\}. \qquad (22)$$

The growth rate (22) exhibits two distinguishing features as compared with that obtained previously [21] under the assumption that the electron distribution function is close to an anisotropic bi-Maxwellian. First, the growth rate contains a small correction coming from the electron collision frequency. Second, in Eq. (22) the coefficient in front of the first addendum inside the curly brackets is twice as large. The growth rate is maximized by the value $k=k_m=\sqrt{2}v_E\omega_L/3v_Tc$ being

$$\gamma_m=\frac{8}{27\sqrt{\pi}}\omega_L\frac{v_E^3}{cv_T^2}\left(1-\frac{\pi\sqrt{3\pi}}{4}\frac{\nu}{\omega_L}\frac{cv_T}{v_Ev_L}\right). \tag{23}$$

Concerning Eqs. (22) and (23), the following must be noted. To obtain Eq. (22) in Eq. (21) the integration over velocities must be carried out. In the integrals not containing the collision frequency $\nu(v)$ the main contribution is given by thermal electrons with velocities $v\simeq v_T$. On the other hand, in the integral containing $\nu(v)$ the main contribution is given by electrons with velocities $v\simeq v_L$. We remind the reader that in deriving Eq. (21) the inequality $kv\gg\nu(v)$ was assumed [see Eq. (19)]. For electrons with $v\approx v_L$, this inequality takes the form $kv_L\gg\nu v_T^3/v_L^3\gg\nu$. It means that within the adopted assumption, the contribution of collisions to the growth rate (22), (23) is relatively small. It must also be observed that the second inequality in Eq. (19), implying a small growth rate, is automatically fulfilled, as γ_m (23) is smaller than k_mv_T by the factor $v_E^2/v_T^2\ll 1$.

Let us now consider the growth rate in the case where the Langdon parameter α (7) is much greater than unity. In this limit and for time intervals longer than the characteristic time of electron temperature doubling $t\gtrsim v_T^2/\nu v_E^2$, the electron distribution has the form (10). Using the distribution (10) from Eq. (21) we obtain

$$\gamma=\frac{4\sqrt{5}}{3\pi}\frac{\left[\Gamma\left(\frac{8}{5}\right)\right]^{3/2}}{\Gamma\left(\frac{7}{5}\right)}kv_T\left\{\frac{2}{5\left[\Gamma\left(\frac{8}{5}\right)\right]^2}\right.$$
$$\left.\times\frac{v_E^2}{v_T^2}\left[\Gamma\left(\frac{6}{5}\right)-\frac{\pi\Gamma\left(\frac{7}{5}\right)}{40\left[\Gamma\left(\frac{8}{5}\right)\right]^2}\frac{\nu}{kv_T}\right]-\frac{k^2c^2}{\omega_L^2}\right\}. \tag{24}$$

In this case, too, the influence of collisions is small and reduces to a decreasing of the growth rate by the k-independent quantity $(\nu v_E^2/v_T^2)/15\sqrt{5}\Gamma^{5/2}(8/5)$. The maximum growth rate value is reached at

$$k_m=\frac{\omega_L}{c}\frac{v_E}{v_T}\left[\frac{2\Gamma\left(\frac{6}{5}\right)}{15\left[\Gamma\left(\frac{8}{5}\right)\right]^2}\right]^{1/2} \tag{25}$$

which is smaller by a factor of 1.2 compared with the case where the electron distribution is close to Maxwellian (8). As a consequence, the growth rate maximum value is also smaller:

$$\gamma_m=\frac{16\sqrt{2}\left[\Gamma\left(\frac{6}{5}\right)\right]^{1/2}}{45\pi\sqrt{3}\Gamma\left(\frac{7}{5}\right)\left[\Gamma\left(\frac{8}{5}\right)\right]^{3/2}}\omega_L\frac{v_E^3}{cv_T^2}$$
$$\times\left[\Gamma\left(\frac{6}{5}\right)-\frac{3\pi\Gamma\left(\frac{7}{5}\right)}{80\sqrt{2}\Gamma\left(\frac{8}{5}\right)\left[\Gamma\left(\frac{6}{5}\right)\right]^{1/2}}\frac{\nu c}{\omega_Lv_E}\right]. \tag{26}$$

In the limit where the small collisional corrections are neglected, the growth rate (26) is smaller than Eq. (23) by a factor of 1.5. In turn, it implies that the Langdon distribution (10), formed as a result of electron heating due to inverse bremsstrahlung, decreases the growth rate of the Weibel instability.

IV. AMPLIFICATION OF THE VORTEX MAGNETIC FIELD UNDER WEAK RADIATION FIELD ACTION

In this section we consider the possibility of vortex magnetic field amplification due to the development of the Weibel instability. Let us confine the discussion to the linear stage of the instability, when the time evolution of the magnetic field energy density $W(k,t)=B^2(k,t)/8\pi$ is described by the equation

$$\frac{d}{dt}W(k,t)=2\gamma(k,t)W(k,t), \tag{27}$$

where the instability growth rate $\gamma(k,t)$ evolves with time according to Eqs. (6), (9), (11), (21), (22), (24). We discuss first the case where the Langdon parameter (7) is small and the electron distribution is given by Eq. (8). The quasistationary distribution (8) is established in a time of the order $v_T^2/v_E^2\nu(v_L)\approx Z/\nu$. The use of this distribution is meaningful for times $\gtrsim Z/\nu$. Besides, the relations (21), (22), (24) have been obtained using the established value of the nonequilibrium addendum δf (13) to the distribution function, and such a procedure is valid for times greater than the inverse of the electron-ion collision frequency $1/\nu$. Keeping in mind these remarks, we investigate the possibility of magnetic amplification, starting from the time $t_0\gtrsim Z/\nu$. We note that such a time is considerably smaller than the characteristic time needed for electron temperature doubling $\sim(v_T^2/v_E^2\nu)$. To simplify the following discussion, we set $t_0=0$. Then, from Eq. (9) we find

$$v_T=v_T(0)\left(1+\frac{t}{\tau_m}\right)^{1/5}, \tag{28}$$

where the characteristic time of the temperature evolution is given by

$$\tau_m=\frac{9\sqrt{2\pi}}{5}\frac{v_T^2(0)}{v_E^2\nu[v_T(0)]}, \tag{29}$$

with $v_T(0)$ being the electron thermal velocity at $t_0=0$. Further, using the relations (22) and (28) from Eq. (27) we arrive at

$$W(k,t)=W(k,0)\exp[R(k,t)], \tag{30}$$

where $W(k,0)$ is the initial magnetic field energy spectral density, while the function $R(k,t)$ describing the amplification is given by

$$\begin{aligned} R(k,t)=&\frac{5\sqrt{2}}{3\sqrt{\pi}}kv_T(0)\tau_m\left\{\frac{v_E^2}{v_T^2(0)}\left[\left(1+\frac{t}{\tau_m}\right)^{4/5}-1\right]\right.\\ &\left.-\frac{k^2c^2}{\omega_L^2}\left[\left(1+\frac{t}{\tau_m}\right)^{6/5}-1\right]\right\}\\ &-\pi\sqrt{24\pi}\,\frac{v_T(0)}{v_L(0)}\left[\left(1+\frac{t}{\tau_m}\right)^{2/15}-1\right], \end{aligned} \tag{31}$$

$v_L(0)=[\sqrt{\pi/8}Zv_E^2v_T(0)]^{1/3}<v_T(0)$. The function $R(k,t)$ reaches its maximum value at the wave number

$$\begin{aligned} k_{mR}(t)=&\frac{\omega_L v_E}{\sqrt{3}cv_T(0)}\left[\left(1+\frac{t}{\tau_m}\right)^{4/5}-1\right]^{1/2}\\ &\times\left[\left(1+\frac{t}{\tau_m}\right)^{6/5}-1\right]^{-1/2}, \end{aligned} \tag{32}$$

which decreases with time as the electrons get heated. When it occurs, the maximum value of the function $R(k,t)$ itself increases:

$$\begin{aligned} R_m(t)=&\frac{10\sqrt{2}}{9\sqrt{3\pi}}\omega_L\tau_m\frac{v_E^3}{cv_T^2(0)}\left[\left(1+\frac{t}{\tau_m}\right)^{4/5}-1\right]^{3/2}\\ &\times\left[\left(1+\frac{t}{\tau_m}\right)^{6/5}-1\right]^{-1/2}-\pi\sqrt{24\pi}\,\frac{v_T(0)}{v_L(0)}\\ &\times\left[\left(1+\frac{t}{\tau_m}\right)^{2/15}-1\right]. \end{aligned} \tag{33}$$

In Eq. (33) the last term describes the small influence on $R_m(t)$ of the collisions. With time, due to the decrease of the electron-ion collision frequency, the contribution to $R_m(t)$ value by the collisions becomes increasingly smaller. However, even weak collisions may have a strong influence on the efficiency of magnetic field amplification. In fact, at $v_T(0)>v_L(0)$, as soon as $t\gtrsim 0.3\tau_m v_L(0)/v_T(0)$, the contribution to the function $R(k,t)$, Eqs. (31) and (33), due to the collisions becomes larger than unity in absolute value. As can be seen from Eqs. (30) and (31), for the above times the decrease of $R(k,t)$ due to collisions yields a significant weakening of the magnetic field amplification efficiency. This conclusion is in agreement with the finding of Ref. [20], where it has been shown that the collisions decrease the energy density of the magnetic field generated as consequence of the Weibel instability. At the same time there is an essential difference. In the conditions under consideration the influence of collisions on the Weibel instability growth rate is controlled by subthermal electrons having large collision frequencies.

According to Eqs. (30) and (33), it is meaningful to speak about magnetic field amplification provided $R_m(t)\gg 1$. In such conditions, for the magnetic field energy density approximately one has

$$\frac{B^2}{8\pi}=\int\frac{dk}{2\pi}W(k,t)\simeq\frac{W(k_{mR},0)}{\sqrt{2\pi|R_m''(t)|}}\exp[R_m(t)], \tag{34}$$

where $R_m(t)$ has the form (33), while the second derivative with respect to k is equal to

$$\begin{aligned} R_m''(t)=&-\frac{10\sqrt{2}}{\sqrt{3\pi}}v_E\tau_m\frac{c}{\omega_L}\left[\left(1+\frac{t}{\tau_m}\right)^{4/5}-1\right]^{1/2}\\ &\times\left[\left(1+\frac{t}{\tau_m}\right)^{6/5}-1\right]^{1/2}. \end{aligned} \tag{35}$$

In integrating Eq. (34) over k it is assumed that the function $W(k,0)$ does not have strong singularities and weakly varies inside a small region around k_{mR} with dimension $\Delta k \simeq\sqrt{2/|R_m''(t)|}\ll k_{mR}$. As can be seen from Eqs. (33) and (34), under the conditions

$$\frac{v_T(0)}{\sqrt{Z}}>v_E\gg c\,\frac{\nu[v_T(0)]}{\omega_L}\,\frac{v_T(0)}{v_L(0)}, \tag{36}$$

which may occur in a sufficiently hot underdense plasma, the magnetic field amplification starts at times $t<\tau_m$, when it is possible to neglect the electron heating. On the contrary, if the right-hand side of the inequality (36) is violated, then the magnetic field amplification is possible only in the presence of electron heating. Such a possibility implies the use of high frequency radiation pulses of relatively large duration.

V. MAGNETIC FIELD AMPLIFICATION IN THE CASE OF THE LANGDON DISTRIBUTION FUNCTION

In the case where the amplitude v_E of the electron quiver velocity in the presence of a high frequency field satisfies the inequalities

$$v_T>v_E>\frac{v_T}{\sqrt{Z}}, \tag{37}$$

the electromagnetic heating due to inverse bremsstrahlung is followed by the formation of a nonequilibrium distribution function of the type (10). In the case where an initially Maxwellian electron distribution $f_m(v)=n(2\pi)^{-3/2}v_T^{-3}(0)\exp[-v^2/2v_T^2(0)]$ rearranges into the self-similar distribution (10), the magnetic field amplification is described by Eqs. (21) and (27) with the addition of the kinetic equation (6), in which the electron-electron collision integral is dropped. Let us now consider such a case. Introducing the new variables

$$u=\frac{v^2}{2v_T^2(0)},\quad \tau=\frac{v_E^2}{6v_T^2(0)}\nu[v_T(0)]t \tag{38}$$

and the function

$$F(u,\tau)=(2\pi)^{3/2}v_T^3(0)\frac{f_0}{n}, \tag{39}$$

for $F(u,t)$ we obtain the equation

$$\frac{\partial}{\partial\tau}F(u,\tau)=\frac{1}{\sqrt{2u}}\frac{\partial^2}{\partial u^2}F(u,\tau) \tag{40}$$

with the following boundary and initial conditions:

$$\frac{\partial}{\partial u}F(u,\tau)|_{u=0}=0,\quad F(u\to\infty,\tau)=0, \tag{41}$$

$$F(u,\tau=0)=\exp(-u). \tag{42}$$

According to Eqs. (21), (27), the solution to Eqs. (40)–(42) allows one to find the function $R(x,\tau)$, characterizing the magnetic field amplification

$$w(k,\tau)=\frac{W(k,\tau)}{W(k,0)}=\exp[R(x,\tau)], \tag{43}$$

where $x=kcv_T(0)/\omega_L v_E$. In the new variables the function $R(x,\tau)$ has the form

$$R(x,\tau)=\sqrt{2}\int_0^\tau d\tau'\left[\int_0^\infty du\,u\frac{\partial}{\partial u}F(u,\tau')\right]^{-1}\times\left\{\frac{12}{\sqrt{\pi}}xA\left[\frac{4}{3\sqrt{\pi}}\int_0^\infty du\sqrt{u}\,\frac{\partial}{\partial u}F(u,\tau')+x^2\right]-\int_0^\infty du\,u^{-3/2}\frac{\partial}{\partial u}F(u,\tau')\right\}. \tag{44}$$

Equations (41)–(44) contains only the parameter

$$A=\frac{\omega_L v_E}{c\nu[v_T(0)]}. \tag{45}$$

When $A\gg 1$ significant magnetic field amplification takes place at $\tau<1$, i.e., for times smaller than both the characteristic time required for the establishment of the self-similar distribution (10) and the time of initial temperature doubling. When $A\gg 1$, to define the function $w(x,\tau)$ (43) up to times $\tau\gtrsim 1$, it is useful to solve Eqs. (40)–(42) numerically. In Fig. 1 the function $F(u,\tau)$ is plotted for few time moments τ. The results of the numerical evaluation of function $w(x,\tau)$ for $A=5$ are shown in Fig. 2, where it can be seen that $w(x,\tau)$ has a strong maximum at a well-defined value $x_m(\tau)$ decreasing with time. At $x=x_m(\tau)$ the function $w(x_m,\tau)$ shows fast increase and at $\tau=0.3,0.6,1.$ its values are $w(x_m,\tau)=8,50,350$.

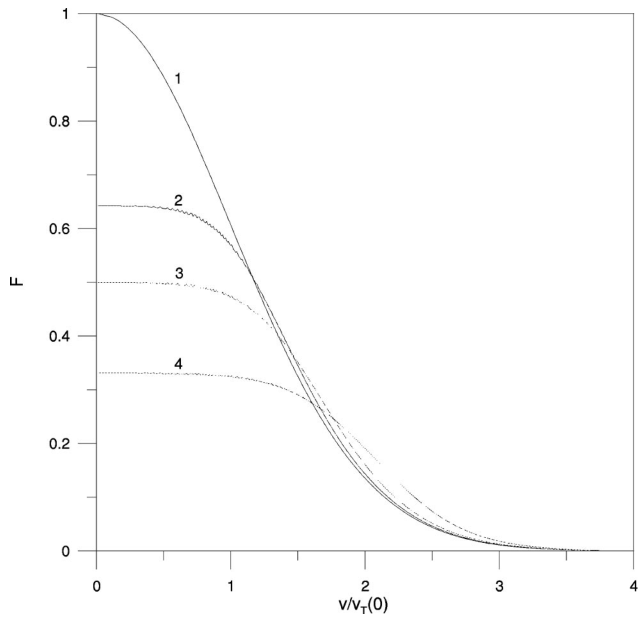

FIG. 1. Electron distribution function evolution due to inverse bremsstrahlung heating. Curves correspond to different time moments τ: $1-0$, $2-0.1$, $3-0.3$, $4-1$. $\tau=v_E^2\nu[v_T(0)]t/6v_T^2(0)$, $v_T(0)$ is the initial electron thermal velocity, v_E is the electron quiver velocity, $\nu[v_T(0)]$ is the electron-ion collision frequency.

Further description of the magnetic field amplification at $\tau\gtrsim 1$ may be carried out using the self-similar distribution (10) and (11) and the corresponding instability growth rate (24). These equations describe magnetic field amplification for $A<1$ as well. This possibility stems from the circumstance that, when $A<1$, we may neglect variations of spon-

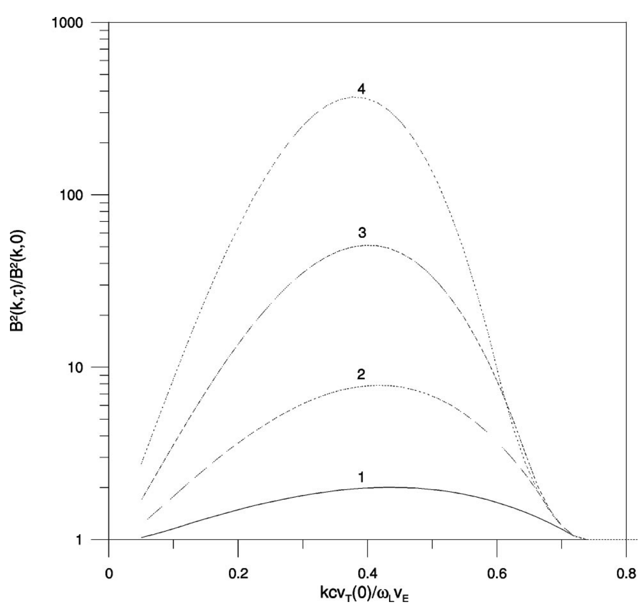

FIG. 2. Relative amplification of spectral magnetic energy density in the linear stage of Weibel instability with $A=\omega_L v_E/c\nu[v_T(0)]=5$; ω_L is the plasma frequency. Curves correspond to different time moments τ: $1-0.1$, $2-0.3$, $3-0.6$, $4-1$. Other notations are the same as in Fig. 1.

taneous magnetic field in the time when the self-similar electron distribution establishes. According to Eq. (11), when the plasma electrons are described by the self-similar distribution, their effective thermal velocity increase with time is given by Eq. (28),

$$v_T = v_T(0)\left(1+\frac{t}{\tau_L}\right)^{1/5}, \tag{46}$$

with, however, a different characteristic time constant

$$\tau_L = \frac{6\sqrt{5}\,\Gamma^{3/2}\left(\frac{8}{5}\right)v_T^2(0)}{v_E^2\,\nu[v_T(0)]}, \tag{47}$$

which is greater than τ_m (29) by a factor of 2.2. Having in mind these relations, from Eqs. (24) and (27) for the function $w_L(k,t)$ describing the magnetic field amplification we find

$$w_L(k,t) = \exp[R_L(k,t)], \tag{48}$$

$$R_L(k,t) = \frac{20\sqrt{5}}{9\pi}\,\frac{\Gamma^{3/2}\left(\frac{8}{5}\right)}{\Gamma\left(\frac{7}{5}\right)}\,k v_T(0)\,\tau_L\left\{\frac{3\Gamma\left(\frac{6}{5}\right)}{5\Gamma^2\left(\frac{8}{5}\right)}\,\frac{v_E^2}{v_T^2(0)}\times\left[\left(1+\frac{t}{\tau_L}\right)^{4/5}-1\right]-\frac{k^2c^2}{\omega_L^2}\left[\left(1+\frac{t}{\tau_L}\right)^{6/5}-1\right]\right\} -\frac{4}{5}\ln\left(1+\frac{t}{\tau_L}\right). \tag{49}$$

When the wave number k is

$$k_{mL}(t) = \left[\frac{\Gamma\left(\frac{6}{5}\right)}{5\Gamma^2\left(\frac{8}{5}\right)}\right]^{1/2}\frac{\omega_L v_E}{c v_T(0)}\left[\left(1+\frac{t}{\tau_L}\right)^{4/5}-1\right]^{1/2}\times\left[\left(1+\frac{t}{\tau_L}\right)^{6/5}-1\right]^{-1/2}, \tag{50}$$

the function $R_L(k,t)$ reaches its maximum, which increases with the electron heating. The corresponding relative amplification of the magnetic field energy spectral density is

$$w_{mL}(t) = \exp[R_{mL}(t)] = \left(1+\frac{t}{\tau_L}\right)^{-4/5}\times\exp\left\{\frac{16}{3\pi}\sqrt{5}\,\frac{\Gamma^{3/2}\left(\frac{6}{5}\right)}{\Gamma\left(\frac{7}{5}\right)\Gamma^{3/2}\left(\frac{8}{5}\right)}\times A\left[\left(1+\frac{t}{\tau_L}\right)^{4/5}-1\right]^{3/2}\left[\left(1+\frac{t}{\tau_L}\right)^{6/5}-1\right]^{-1/2}\right\}. \tag{51}$$

The factor in front of the exponent in Eq. (51) initiated by the influence of collisions on the instability growth rate and is responsible for the weakening of the magnetic field amplification by the value $(1+t/\tau_L)^{4/5}$. According to Eq. (51), if $A \ll 1$, amplification is possible for $t \gg \tau_L$. In this connection, it is worth noting that expression (51) is valid provided the right-hand side of the inequality (37) holds, guaranteeing a weak influence of electron-electron collisions on the distribution function. In other words, the relation (51) is valid only for times obeying the condition $t < \tau_L[Zv_E^2/v_T^2(0)]^{5/2}$.

VI. CONCLUSION

We have shown that it is possible to amplify spontaneous magnetic fields in a plasma heated via inverse bremsstrahlung by a relatively weak high frequency radiation field. Let us now discuss in more detail the conditions under which the described process may take place. As an example we consider a sufficiently hot plasma with an electron effective temperature $T \approx 1$ keV, $Z = 10$ and an electron density $n \approx 10^{20}$ cm^{-3}. The plasma interacts with a radiation field with $\omega = 2\,10^{15}\,\mathrm{sec}^{-1}$ and flux density $I = cE^2/8\pi \approx 3\times 10^{14}$ W/cm^2. With these parameters $\alpha = Zv_E^2/v_T^2 \leqslant 1$ and the distribution function of the bulk electron velocities may be approximated by a Maxwellian. Further, for the time τ_m, giving the scale of electron temperature variation, and for the parameter A, characterizing the magnetic field amplification, we have $\tau_m \approx 15$ psec and $A \approx 3.3$. As may be seen from relation (33), the magnetic field amplification starts at $t \approx \tau_m \approx 15$ psec, and at $t \approx 150$ psec, when the electron temperature increases by a factor 2.5, the magnetic field energy density increases by a factor of 10^6. Of course, if the laser field pulse duration τ_p is smaller than 150 psec the amplification will be weaker; or even absent if $\tau_p \leqslant 15$ ps. Increasing the radiation flux density up to $I \approx 1.4\,10^{15}$ W/cm^2, with unchanged plasma parameters, will give $v_T \approx \sqrt{2}v_E$, $\alpha \approx 5$, $A \approx 7$ and Weibel instability develops in the regime when the Langdon distribution is formed with the time constant $\tau_L \approx 8$ psec. In such a case, as may be seen from Eq. (51), a magnetic field amplification much stronger than predicted above takes place even before the self-similar distribution (10) is established.

In the case where the spontaneous magnetic field level is not anomalously small, a rapid development of the Weibel instability yields the generation of such magnetic fields, which are required for a correct description to take into account nonlinear effects of stabilization [19]. Following Ref. [19] we may give an estimate of the magnetic field strength generated in the instability stage, when nonlinear effects become important.

According to Ref. [19], in the nonlinear stage the magnetic field energy density $B_m^2/8\pi$ is about 10% of the anisotropy degree of the electron pressure Δp. In a high-frequency field one may assume $\Delta p \approx n m v_E^2/3$. Then, with $n \approx 10^{20}$ cm^{-3} and $I \approx 1.4\,10^{15}$ W/cm^2, one has $B_m \approx 0.25$ MGs. If the laser field pulse duration is greater than the time required to enter into the nonlinear state, then the magnetic field B_m will further evolve with time. Of course, for a correct description of this instability stage the nonlinear effects must be duly taken into account. At the same time, even for a magnetic field of about a quarter of a megaGauss the electron Larmor frequency is $\Omega = |eB|/mc \approx 4.4 \times 10^{12}\,\text{sec}^{-1}$ and is larger than the electron effective collision frequency $\nu \approx 2.5\,10^{12}\,\text{sec}^{-1}$. When $\Omega \gtrsim \nu$, the properties of the electron transport depend on the generated magnetic fields. Strong quasistationary magnetic fields affect, to a considerable extent, the low-frequency spectrum of plasma radiation as well. The quoted effects are important in their own right, and worthy of a separate investigation, to make more complete the theory of magnetic field generation due to inverse bremsstrahlung. Another perspective direction of investigation, concerning the Weibel instability in a plasma interacting with a strong laser field, is provided by the elucidation of the conditions for which it is required to take into account both the relativistic effects and the role of the plasma and the radiation inhomogeneity. With it we have in mind the transition domains and the need to have relatively simple and transparent treatments, allowing one to properly understand the role of the several process parameters. The interest in this kind of investigation is directly connected to several new experiments, in which an intense ultrashort laser pulse interacts with plasmas formed by the laser pulse itself through ionization. These plasmas are intrinsically inhomogeneous. Further, an important peculiarity of such laser-produced plasmas is that they exhibit an anisotropic photoelectrons velocity distribution, and such anisotropy acts as a source of the Weibel instability. Thus, it is evident that the generation of quasistationary magnetic fields in a plasma may be interwoven with several other phenomena that need to be properly understood. In conclusion, we consider that the results found in the present work and concerning the behavior of the Weibel instability in a plasma interacting with a radiation field of moderate intensity may be looked at as a useful starting point for a new direction of investigations concerning the important issue of the nonequilibrium properties of laser plasmas.

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